Optimization for Quantum Information Science Problems

Jeffrey Larson

Argonne National Laboratory

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  - ? Maximizing quantum Fisher information

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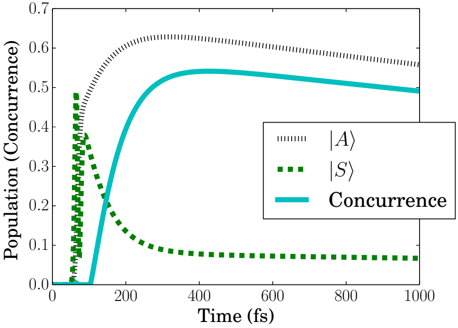

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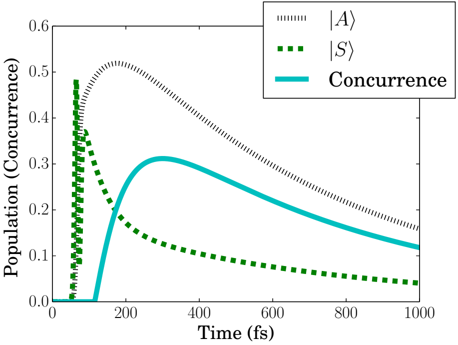

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$$C_{ij} = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\},$$

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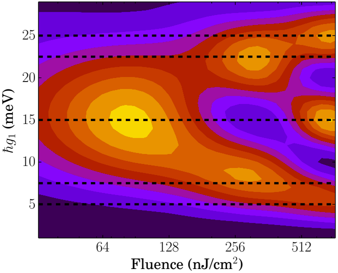

Otten, Larson, Min, Wild, Pelton, Gray. Origins and optimization of entanglement in plasmonically coupled quantum dots. Physical Review A, 2016

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- There is a computable definition for even numbers of qubits which could be used.
- Higher dimensional entanglement is less well understood theoretically, but there are some special states would be interesting to try and create.

Outline

Real-world circuit fidelities

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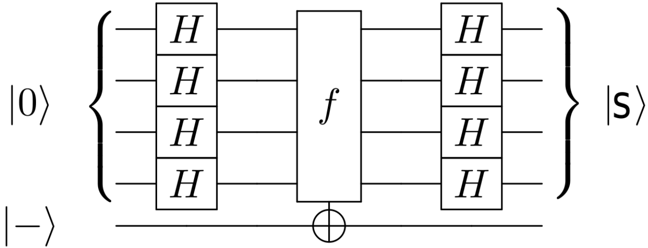

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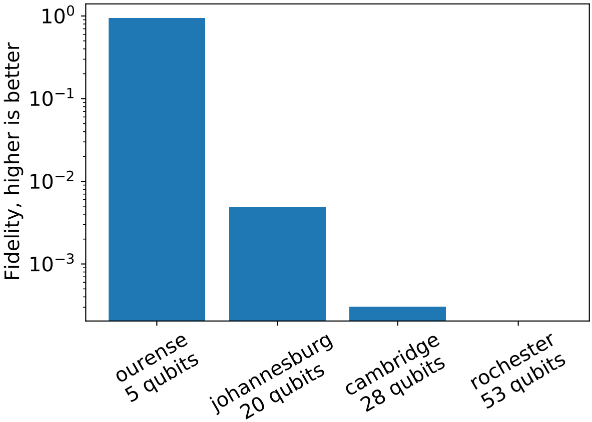

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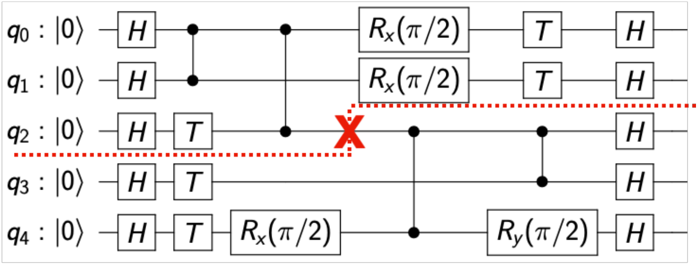
- We consider a hybrid classical/quantum computing approach
- Cuts large quantum circuits into smaller subcircuits
- Classical post-processing can then reconstruct the output of the original circuit.



Modeling variables

$$y_{v,c} \equiv \begin{cases} 1 & \text{if vertex } v \text{ is in subcircuit } c \\ 0 & \text{otherwise} \end{cases}, \ \forall v \in V, \forall c \in C$$

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The number of qubits required to run a subcircuit is the sum of:

- The number of original input qubits
- The number of initialization qubits induced by cutting

Modeling variables

- The number of original input qubits is $\alpha_c \equiv \sum_{v \in V} w_v \times y_{v,c}, \forall c \in C$, where $w_v \in \{0, 1, 2\}$ is the number of original input qubits directly connected to $v \in V$.

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$$O_c \equiv \sum_{e:(e_a,e_b) \in E} x_{e,c} \times y_{e_a,c}, \forall c \in C.$$

Consequently, the number of qubits in a subcircuit that contributes to the final measurement of the original uncut circuit is

$$f_c \equiv \alpha_c + \rho_c - O_c, \forall c \in C.$$

Constraints

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- Some symmetry-breaking constraints

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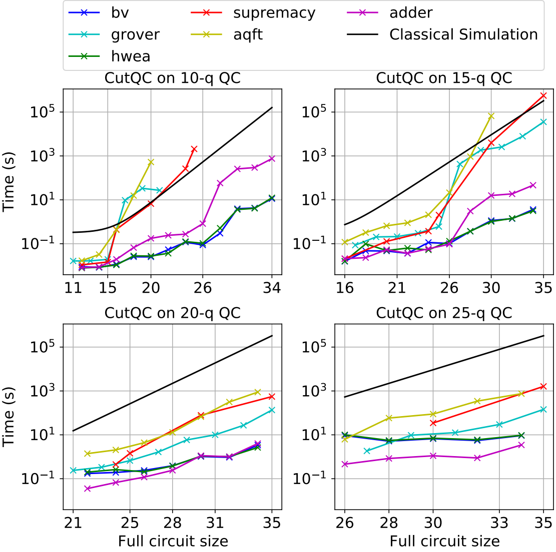
- The objective function for the MIP cut searcher is reconstruction time estimator

$$L \equiv 4^K \sum_{c=2}^{n_C} \prod_{i=1}^{c} 2^{f_i},$$

which captures cost of building the full 2^n probabilities for a n-qubit uncut circuit

Results



Tang et al. "CutQC: Using Small Quantum Computers for Large Quantum Circuit Evaluations." ASPLOS21

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Scaling up quantum devices is a challenge

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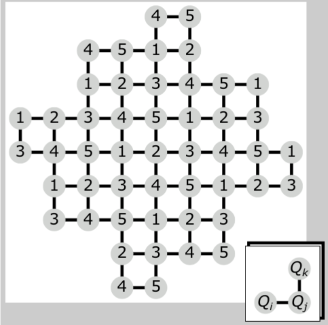
- Scaling fixed-frequency architectures requires precise relative frequency requirements.
- Want to avoid collisions in frequencies.



Hertzberg et al., https://arxiv.org/pdf/2009.00781.pdf

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- The yield measure the number of a potentially valid quantum processor
- Chips are fabricated in batches, and they want to have at least one valid chip per batch.

Frequency collisions can take a variety of forms

- f_i avoid the $0 \mapsto 1$ transitions of j:

$$|f_i - f_j| \geq \delta_1 \qquad \forall (i, j) \in E$$

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$$|2f_i + \alpha_i - f_k - f_j| \geq \delta_7$$
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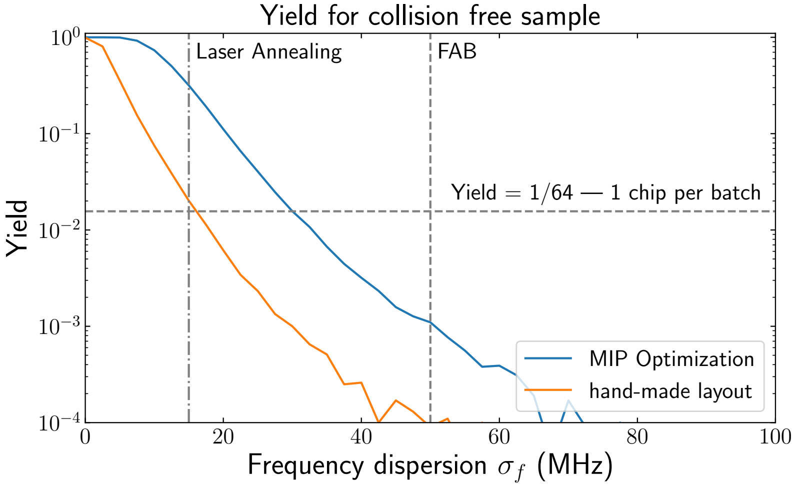
with $\delta_i \geq \bar{\delta}_i$.

Two solutions on 6-node ring



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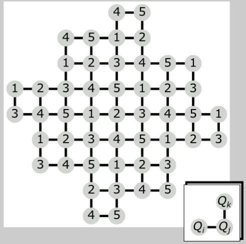
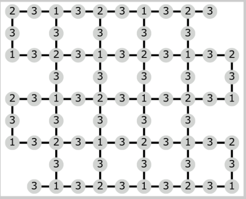
- Trying to optimize the connectivity in the graph. (For now, just assigning frequencies to a given architecture.)





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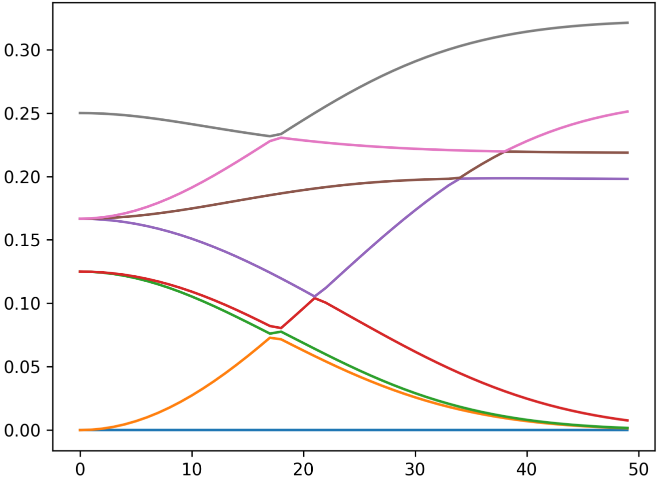
- For large N, computing the QFI can be prohibitively difficult. Many papers maximize (upper) bounds of QFI

Mathematical fun

Example with $N = 8$. Take a starting point x_0 and a random direction d. Compute eigen-decomposition for $\rho(x_0 + \alpha_i d)$ and plot eigenvalues



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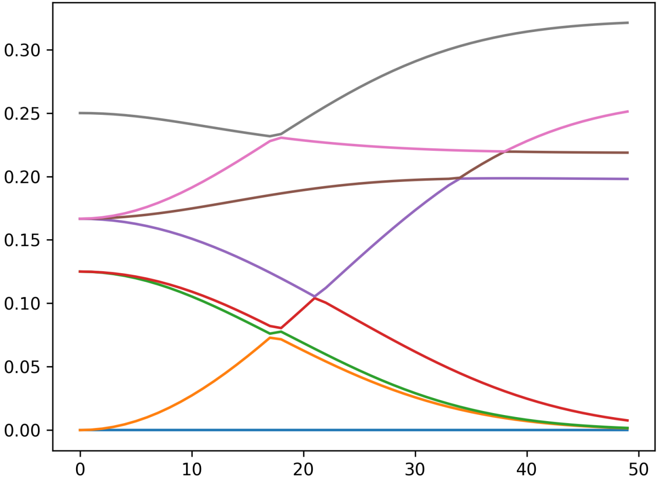

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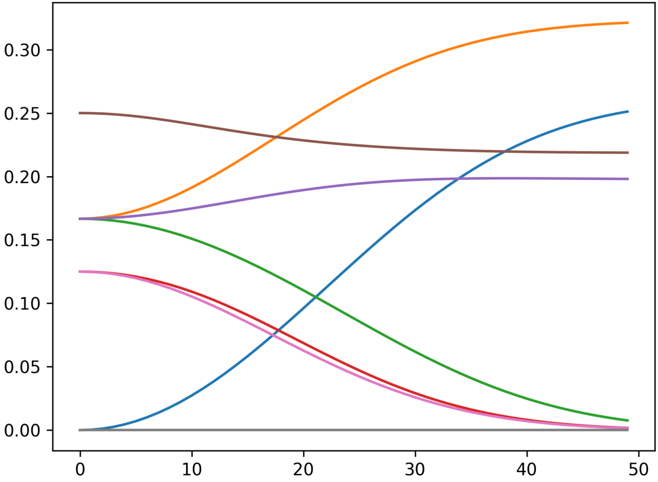

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- Some H/ρ pairs may have analytic forms for their eigenpairs (but we really need cross-products to be "nice"). What could we do in that case?
- Short time optimization vs. steady-state optimization. Are we trying to optimize for sensing at some time t or at infinity?

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  - Has known derivative or subdifferential
  - Relatively cheap to evaluate
- $F \colon \mathbb{R}^n \to \mathbb{R}^p$ is relatively unknown
  - Based on a simulation
  - Relatively expensive to evaluate
  - Stochastic

Use knowledge of h to use fewer calls to F.

Contact

Thanks for listening! Questions?

jmlarson@anl.gov

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